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A STUDY OF THE TWO-CONTROL OPERATION OF AN AIRPLANE

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SUMMARY

The two-control operation of a conventional airplane is treated by means of the theory of disturbed motions. The consequences of this method of control are studied with regard to the stability of the airplane in its unconstrained components of motion and the movements set up during turn maneuvers.

It is found that the motion of a conventional airplane is more stable when an arbitrary kinematic constraint is imposed in banking than when such constraint is imposed in yawing. Several hypothetical assumptions of piloting procedure, each of which is considered to represent a component of the actual procedure, are studied. Different means of two-control operation are also discussed and it is concluded that a reliable rolling-moment control that does not give the usual adverse secondary yawing moment should be most satisfactory. Several special modifications intended to make the airplane more suitable for two-control operation are also discussed, and it is found that relatively great weathercock stability (N_z) would be desirable.

INTRODUCTION

A number of flights have been made with airplanes utilizing both the aileron-elevator and the elevator-rudder combinations for two-control operation. Some question exists as to which of these modes of operation is likely to prove the better and also whether either of them is capable of affording the controllability requisite to safety in flight. Such questions must, of course, be eventually decided by experience, no mathematical analysis being sufficiently broad to deal with all aspects of the problem. It is believed, nevertheless, that certain conceptions gained from an analysis of the problem may be useful in furthering development along these lines.

One of the purposes of the present work was to ascertain on theoretical grounds which of the two possible modes of operation was more likely to prove satisfactory. It was also desired to find what changes might be effected in a conventional airplane to make it more suitable for two-control operation.

The analysis of the various dynamical problems that arise makes use of many concepts that are discussed at length in reference 1. The treatment of airplane motion as a problem of dynamics is based primarily on

the assumptions of the theory of airplane stability as developed by Bryan and others; for the elucidation of this theory the reader is referred to text books on aeronautics.

MATHEMATICAL TREATMENT OF CONTROLLED MOTION

The motion of an airplane with adequate control about its three axes may, in one sense, be regarded as a purely constrained motion. From this point of view, the act of piloting the airplane must be considered to be the use of the available control means for overcoming the inherent aerodynamic and inertial reactions of the airplane, causing it to follow a more or less definitely constrained motion induced by the controls. The natural oscillation and damping of the free motion of the airplane do not appear, then, in the controlled motion because the pilot has accommodated his use of the available control to the governing of these inherent tendencies. Accordingly the stability or instability of the airplane will be apparent only in the requisite use of the controls to perform a given maneuver.

It has been found by experience that the lateral-stability characteristics of an ordinary airplane are such that it is feasible to abandon one of the direct constraints of the lateral motion in ordinary flight maneuvers. All lateral maneuvers that are to be performed with a minimum of sideslipping or sidewise acceleration require a definite coordination between the banking and yawing motions; it appears that a conventional airplane will naturally tend to fulfill this requisite relation in greater or less degree, on account of the inherent stability, even when one of the lateral controls is abandoned.

Under the conditions of two-control operation the motion of the airplane cannot be considered as an entirely constrained motion. The pilot of such a machine can exercise direct constraint in only one of the three components of lateral movement and must depend on the natural tendencies of the airplane for the requisite coordination of the other motions. In order to show this coordination the airplane need not be entirely stable with all controls released, but it is imperative that there be satisfactory stability in those components in which the machine is unconstrained. Thus, if an

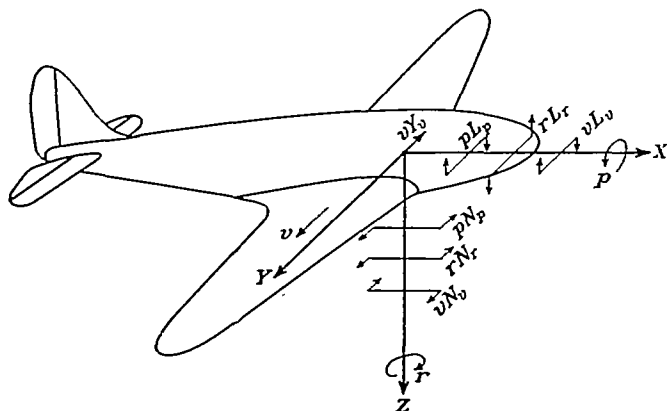
airplane is to be controlled by the ailerons and elevator alone, it must be satisfactorily stable in combined yawing and sideslipping, in which it is free; if control is by rudder and elevator, corresponding stability in combined banking and sideslipping is necessary.

If the controls are considered to impress constraints in those components of motion in which they operate directly, the movements of a two-control airplane may be studied by the method of forced oscillations. Thus, if the airplane controlled by ailerons is caused to follow a definite course in banking, in which it is considered to be constrained, this motion will impress disturbing forces and couples leading indirectly to yawing and sideslipping motions. The yawing and sideslipping motions must, however, be considered to be unconstrained and to be conditioned by the natural stability of the machine as well as by the impressed disturbances.

The disturbing forces or couples impressed in those components in which the airplane is unconstrained are caused by the constrained movements and are considered proportional to them. The factors of proportionality are simply the appropriate stability derivatives of the airplane. Thus, if the machine is constrained to follow a definite sequence of rolling motions by the application of a suitable control moment, a disturbing acceleration in yawing that is proportional to the given rate of rolling at each instant will be impressed, namely:

$$\text{impressed } \frac{dr}{dt} = p \times N_r$$

In order to express the foregoing ideas definitely it will be necessary to resort to mathematical treatment of the motions. It is convenient for this purpose to choose a set of axes rigidly fixed in the airplane at its center of gravity and inclined at the angle of attack α , so that the X axis points into the direction of the relative wind in steady flight at the specified lift coefficient. The following notation and diagram define the quantities used in the subsequent equations.



U_0 , forward (X -wise) velocity in steady flight.

p , rolling component of angular velocity.

r , yawing component of angular velocity.

v , component of flight velocity along Y axis (sideslip).

φ , angle of bank (relative to gravity).

β , angle of sideslip v/U_0 , approximately.

δ , angle of rudder or aileron deflection.

Y , force component along the direction of the Y axis.

L , rolling-moment component.

N , yawing-moment component.

$\delta L_\delta = L/mk_x^2$, $\delta N_\delta = N/mk_z^2$, Control moments per unit moment of inertia of airplane.

Y_r , Stability derivatives in terms of unit mass or moment of inertia of airplane, thus:

$$Y_v = \frac{\partial Y}{\partial v} / m$$

$$L_r = \frac{\partial L}{\partial r} / mk_x^2, \text{ etc.}$$

A number of secondary considerations will be neglected in the mathematical analysis of the problems to make the mathematical expressions as simple as possible and because it is not considered important to secure exact numerical results for studying the general problem. For these approximate calculations the lateral and longitudinal motions of the airplane will be considered separable during turning flight. A check of the maximum gyroscopic couples encountered shows that they are negligible for the present study, although it is probable that the longitudinal and lateral oscillations in turning flight can be separated for only a relatively short time after the passing of a disturbance. Another assumption made is that the effect of a component torque applied to the airplane is an angular acceleration about the axis of the torque. In general, the angular acceleration does not have the same axis as the applied torque but in the present case the reference axes chosen lie near the assumed principal axes of inertia, and the difference of moments of inertia taken about various axes is not great. In addition, the flight of the airplane is assumed to be horizontal and the speed not to vary appreciably from the average (U_0) in a given case.

According to the previously outlined treatment, the movement of the airplane in at least one of the lateral coordinates will be modified by a constraint. The complete set of three degrees of freedom is not in this case expressed in the usual three simultaneous equations of motion, for this procedure would imply that each component of the motion was affected by the other two, whereas the present problem calls for an independent expression of one of them. Thus, it is assumed that the available control is sufficiently powerful to force any desired motion in the controlled component. When setting up the equations, this

motion will be considered to be given as a function of the time.

It is important to emphasize in the interpretation of the mathematical analysis the practical significance of the assumptions used. The solution of the equations requires that the complete history of the variation of one of the components of the motion (or the control setting) be known beforehand. This variation is not subsequently altered to accommodate the variation of the other motions as would be the case if an intelligent pilot were at the controls. It may be imagined that the pilot has only one degree of attention. Having fixed on a procedure of rolling the airplane, he concentrates on the execution of this alone, paying no attention to the consequences in yawing or sideslipping. It would be feasible to assume that the pilot concentrated his attention on carrying out a predetermined manipulation of the controls, without regard to any of the motions set up. This assumption is, however, considered to be too far removed from actuality to be of much use in analyzing the problem. It would be of more practical interest to assume that the pilot had sufficient skill to enforce a desired motion in every respect, taking no account of the control manipulations. The control manipulations required could then be calculated and an idea of the degree of skill necessary to attain a perfect result could be derived therefrom.

With two-control operation a perfect coordination of the motions is, of course, not possible. If the pilot enforces complete control over one component of the airplane's motion, he must do so at the expense of control in some other component. The residual component is then considered to be free. In practice the pilot can exercise an indirect influence on all lateral motions with only a single lateral control. Hence, it is possible to assume that a skilled pilot could enforce complete control over the yawing motion even though his available control exerted only rolling moments directly. Then the rolling motion must be considered free and not subject to the pilot's attention although his available control operates directly on this motion. Such an assumption obviously cannot give an accurate description of anything occurring in practice. The same is true in some degree of any other assumed procedure that can be mathematically treated. The actual procedure of a pilot is undoubtedly an indeterminate and variable synthesis of such elementary procedures. The study of a single assumption of this nature is therefore incomplete, constituting simply a part in the analysis of the problem.

In order to illustrate the variety of assumptions that may be treated, four equations, containing movements both of the airplane and of the control surface, will be set down:

$$\left. \begin{aligned} \frac{dv}{dt} - g\varphi + rU_0 - vY_s &= 0 \\ \frac{dp}{dt} - pL_p - rL_r - vL_s - \delta L_\delta &= 0 \\ \frac{dr}{dt} - pN_p - rN_r - vN_s - \delta N_\delta &= 0 \\ \frac{d\varphi}{dt} - p &= 0 \end{aligned} \right\} (1)$$

These equations are to be satisfied simultaneously and, since there are more variables than equations, one of the variables must be given in terms of the time to effect a solution. Any assumption of the kind considered may be applied by setting one of the variables equal to a function of t . Thus the equations of motion with an arbitrarily prescribed course in rolling are:

$$\left. \begin{aligned} \frac{dv}{dt} + rU_0 - vY_s &= g\varphi(t) \\ -rL_r - vL_s - \delta L_\delta &= L_r p(t) - \frac{d}{dt} p(t) \\ \frac{dr}{dt} - rN_r - vN_s - \delta N_\delta &= N_r p(t) \end{aligned} \right\} (2)$$

Similarly, if the pilot uses the control to enforce some given motion in yawing, the equations are:

$$\left. \begin{aligned} \frac{dv}{dt} - g\varphi - vY_s &= -U_0 r(t) \\ \frac{dp}{dt} - pL_p - vL_s - \delta L_\delta &= L_r r(t) \\ -pN_p - vN_s - \delta N_\delta &= N_r r(t) - \frac{d}{dt} r(t) \\ \frac{d\varphi}{dt} - p &= 0 \end{aligned} \right\} (3)$$

Solutions of the foregoing differential equations have the general form

$$v, p, \delta, \text{ or } r = (C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \dots + C_n e^{\lambda_n t}) + \zeta(t) \quad (4)$$

This type of solution has two significant components; the part enclosed by parentheses represents the occurrence of the natural oscillations and damping in the resultant motion. If the natural modes of motion are stable, this component will disappear with time and the solution will be represented by $\zeta(t)$. If the impressed disturbance is periodic, the motion will at first be conditioned by the natural period but, if this is damped, will later follow the impressed period in accordance with Herschel's theorem. In these cases the term $\zeta(t)$ may be called the "steady-state solution."

Under the assumed conditions of two-control operation the pilot enforces one component of the motion and relies on the reaction of this motion on the uncontrolled component to induce an appropriate

motion there. As seen in equation (4), this accompanying motion is at first conditioned by the natural oscillations. Obviously for satisfactory two-control operation it is desirable that the natural oscillations in the uncontrolled components quickly die away. It also appears that if any reasonable coordination of the motions is to be obtained the period of the free oscillation must be short compared with the duration of the maneuver.

STABILITY OF A CONVENTIONAL AIRPLANE OPERATED WITH TWO CONTROLS

From the foregoing considerations it is apparent that the airplane must have certain degrees of stability for satisfactory two-control operation. Operation with constraint in yawing calls for stability in combined rolling and sideslipping, whereas operation with rolling constraint requires stability in combined yawing and sideslipping, as indicated by equations (2) and (3). In order to illustrate the degree of stability of a conventional airplane in these motions, data from an assumed average airplane (described in reference 2) have been used and several calculations made for the two cases. The principal characteristics of the assumed airplane are given in the table I.

TABLE I

CHARACTERISTICS OF ASSUMED AVERAGE AIRPLANE

Type.....	Monoplane, 2-passenger.
Gross weight.....	1,600 lb.
Wing area.....	171 sq. ft.
Wing span.....	32 ft.
mk_x^2	1,216 slug-ft. ²
mk_z^2	1,700 slug-ft. ²

Stability derivatives at various lift coefficients:

C_L	L_p	L_r	\dot{L}_p	N_p	N_r	N_y	Y_p
0.35	-5.44	1.11	-0.0544	-0.207	-0.913	0.0368	-0.172
1.0	-3.23	1.88	-0.0415	-0.301	-0.663	0.0231	-0.145
1.8	-2.46	2.51	-0.0461	-0.310	-0.977	0.0221	-0.224

* 5° dihedral.

† Flaps down.

STABILITY WHEN CONSTRAINED IN ROLLING

The stability of the motion of the airplane (or of the movement of the control, δ) when the rolling component is arbitrarily constrained may be calculated from the complementary equations of (2):

$$\left. \begin{aligned} \frac{dv}{dt} + rU_0 - vY_p &= 0 \\ -rL_r - vL_p - \delta L_\delta &= 0 \\ \frac{dr}{dt} - rN_r - vN_p - \delta N_\delta &= 0 \end{aligned} \right\} \quad (5)$$

The complementary equations express only a part of the complete motion. They show the influence of sta-

bility on the manipulations of the control required to enforce the desired constraint in bank as well as the stability of the free yawing and sideslipping oscillations. Whatever rolling motion is assumed, a solution of the complementary equations will appear as a component of the final solution.

The third equation of (5) may be solved for v and the resulting expression substituted into the first equation, etc. The same procedure may be carried out for r or δ ; in either case the so-called "auxiliary" equation is:

$$L_\delta[\lambda^2 - (N_r + Y_p)\lambda + N_rY_p + U_0N_p] + N_\delta[L_r\lambda - L_rY_p - U_0L_p] = 0 \quad (6)$$

The equation is conveniently divided into two parts to show the effects of control rolling and yawing moments. If the rolling motion is constrained by a direct rolling-moment control, the second part of the equation (containing N_δ) is eliminated. Since the first polynomial is a quadratic, its roots are:

$$\lambda = \frac{(N_r + Y_p) \pm \sqrt{(N_r + Y_p)^2 - 4(N_rY_p + N_pU_0)}}{2} \quad (7)$$

If the airplane shows an average degree of weathercock stability ($N_p > 0$), the roots will be conjugate complex numbers and the terms

$$C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

of equation (4) will represent a damped oscillation. If $\lambda_1 = a + ib$ and $\lambda_2 = a - ib$, the period of this oscillation is

$$T_1 = \frac{2\pi}{b} \quad (8)$$

and the time to damp to one-half amplitude:

$$T_2 = -\frac{\log 0.5}{a} = \frac{0.7}{a} \quad (9)$$

provided that a is negative.

Neglecting the first part of equation (6) (containing L_δ) amounts to the assumption that the banking motion is constrained by the application of a rudder control. The solution of this part of the equation alone is:

$$\lambda = Y_p + \frac{U_0 L_p}{L_r} \quad (10)$$

The auxiliary equation thus has only one real root and it is negative, indicating stability. The assumption is that a sidewise disturbance (v) causes the pilot to give the airplane a rate of yawing such that

$$rL_r = -vL_p \quad (11)$$

As L_r is positive, this yawing reduces the sideslip and must then itself be reduced in proportion to prevent rolling, thus resulting in a convergence. This control procedure, although stable and nonoscillatory, represents a more artificial assumption than the control of

the rolling motion by direct rolling moments, for here the pilot in order to check a sudden disturbance must move the airplane as a whole with equal suddenness while with direct control he is only called upon to deflect the control surface suddenly.

Although the motion that occurs when the rolling is controlled—either directly by a variable rolling moment alone or indirectly by a yawing moment—is stable, a control device that gives both rolling and yawing moments in combination may cause instability. Inasmuch as conventional ailerons do give secondary yawing moments, this case is of considerable interest. Denoting the ratio:

$$\frac{N_\delta}{L_\delta} = \kappa$$

where each δ denotes aileron deflection, the following resolution of equation (6) is obtained

$$\lambda^2 - [(N_r - \kappa L_r) + Y_r]\lambda + (N_r - \kappa L_r)Y_r + U_0(N_r - \kappa L_r) = 0 \quad (12)$$

The solution of this equation differs from that of the first component of equation (6) in that the quantities N_r and N_r are replaced by $(N_r - \kappa L_r)$ and $(N_r - \kappa L_r)$, respectively. Thus it is concluded that an effect of a secondary *adverse* yawing moment in an attempted rolling maneuver will be an apparent reduction of both the *weathercock stability* (N_r) and the damping in yawing (N_r).

Calculation shows that the motion becomes unstable when

$$\kappa > \frac{N_r + Y_r}{L_r} \quad (13)$$

or when

$$\kappa > \frac{Y_r N_r + U_0 N_r}{Y_r L_r + U_0 L_r} \quad (14)$$

in negative magnitude. Such instability would indicate that an arbitrary constraint in rolling (such as attempted level flight) could not be maintained by the ailerons alone.

Conventional ailerons give rise to adverse yawing moments in an amount approximately independent of the speed of flight while the rolling moments and stabilizing factors are much reduced at the lower speeds. The result is that the ratio κ approaches the foregoing undesirable magnitude at the highest lift coefficients. It is therefore considered that ordinary ailerons working on a part of the wing surface that sustains a high lift would not be desirable for two-control operation.

Table II lists the results of calculations of the stability indexes of the average airplane in free yawing and sideslipping motions at several lift coefficients. Since these calculations were to be used later in investigating the motions set up during turning maneuvers, a certain increase in the steady-flight speed at a given lift coefficient was assumed. The increase amounted to 7½

percent and the stability derivatives at each lift coefficient were multiplied by this factor.

TABLE II
INDEXES OF STABILITY OF MOTION WITH CONSTRAINT IN ROLLING

	C_L	Roots of stability equation	Period of oscillation	Time to damp ½
			Seconds	Seconds
$\kappa=0$	0.35	$-0.583 \pm 2.50 i$	2.51	1.18
	1.0	$-.435 \pm 1.51 i$	4.16	1.60
	1.8	$-.645 \pm 1.23 i$	5.10	1.08
Adverse yaw $\kappa=-0.15$	1.0	$-.283 \pm 1.31 i$	4.80	2.5
Favorable yaw $\kappa=0.15$	1.0	$-.585 \pm 1.67 i$	3.76	1.2

The combined yawing and sideslipping motion under consideration is, in general, very stable. Further calculations have shown that the stability of the motion when free only in yawing and sideslipping is much greater than the stability of the completely free motion. The oscillations have, in general, a shorter period and greater damping.

STABILITY WHEN CONSTRAINED IN YAWING

Calculation of the stability of the rolling and sideslipping motions when the airplane is constrained in yawing is similar to that given for constraint in banking. Here the complementary equations of (3) are used. The corresponding auxiliary equation is

$$N_\delta[\lambda^3 - (L_p + Y_r)\lambda^2 + L_p Y_r \lambda - g L_r] + L_\delta[-N_p \lambda^2 + N_p Y_r \lambda - g N_r] = 0 \quad (15)$$

The complementary part of the general solution (4) will be of the form

$$p, v, \text{ or } \delta = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3 e^{\lambda_3 t} \quad (16)$$

since there are now three roots. In case the yawing motion is constrained directly by the application of control yawing moments, only the first part of the equation will be in force. Calculation shows that two of the roots will then be of the conjugate complex type previously discussed and that the third root will be very nearly equal to L_p . Table III gives these roots as calculated for the average airplane under conditions similar to those assumed in table II.

TABLE III
STABILITY OF MOTION OF AVERAGE AIRPLANE WITH CONSTRAINT IN YAWING

C_L	Real root	Complex roots	Period of oscillation	Time to damp ½ (complex roots)
			Seconds	Seconds
0.35	-5.90	$-0.064 \pm 0.563 i$	11.2	10.8
1.0	-3.59	$-.019 \pm .636 i$	9.9	36.6
1.8	-2.67	$-.015 \pm .716 i$	8.8	45.3

The fact that the auxiliary equation for the case of free rolling and sideslipping motion with yawing control has roots of such widely different magnitude is an indication that the motion may be separated into distinct modes. The large real root (nearly equal to L_p) indicates the sharp damping of an initial rolling motion and is of such magnitude that the wings may be considered to be in a measure constrained against rolling relatively to the air. A possible rolling motion, however, that will not be appreciably damped consists in rolling about an instantaneous center some distance above the center of gravity of the airplane. For rotation of the airplane as a rigid body about this point the rolling moment due to sideslip will balance the damping of the rolling.¹ The height, \bar{z}_L , of the instantaneous center above the center of gravity is found from:

$$vL_r = -pL_p$$

where

$$v = -pz$$

whence

$$\bar{z}_L = \frac{L_p}{L_r} \quad (17)$$

The mode of motion represented by the small complex roots (table III) thus consists in a swinging oscillation of the airplane about the metacenter \bar{z} as a pendulum suspended from that point. The characteristic roots for the pendulum motion would be

$$\pm \sqrt{-\frac{g}{z}} = \pm i \sqrt{\frac{gL_r}{L_p}} \quad (18)$$

which are seen to be approximate roots of equation (15) ($L_s=0$).

From these considerations it appears that the two-control airplane constrained in yawing with the rudder would be subject to swinging oscillations of long period and slight damping. If the airplane is given an initial angle of sideslip, it will be restrained against banking directly by the relatively great damping in rolling L_p and the banking that occurs will conform nearly to a rotation of the airplane about the metacenter \bar{z}_L . It will be of interest to calculate this height, using the stability derivatives given in table I:

C_L	$\bar{z}_L = \frac{L_p}{L_r}$
0.35	Feet
1.0	100
1.8	92.5
	53

Physical considerations indicate that the damping of this mode of motion is almost entirely dependent on Y_r ; hence, for two-control operation with the rudder, it should be desirable to have a large value of this derivative.

It is possible for the pilot to apply a yawing moment either through the secondary influence of an aileron

control or indirectly by rolling the airplane as a whole. If the latter effect were used to constrain the yawing, the resulting motion would be excessively unstable. Thus, in order to prevent a sidewise disturbance from yawing the airplane ($r=0$), the pilot must execute a roll such that the forward wing is depressed ($pN_r = -vN_s$). This roll provides the occasion for an increase of sideslip due to the bank and requires, in turn, more rapid rolling so that the motion diverges quickly. Secondary aileron yawing moments of either sign moderate this instability and the motion may become stable if the yawing moment is favorable.

These considerations indicate that the pilot could not maintain an exact yawing constraint by the use of ailerons alone. On the other hand, this inability is probably not of great importance since the assumption of piloting procedure is obviously artificial and since the former calculations (stability with constraint in rolling) indicated that, if the ailerons were used to hold the wings level, the free yawing oscillations would be short and quickly damped. (See table II.) Thus it appears that, in order to prevent any yawing whatever during a disturbance, the pilot would have to execute a divergent bank whereas if he merely held the wings level the yawing motion might be unnoticeable. The divergent bank consists in a rotation of the airplane about the metacenter

$$\bar{z}_N = \frac{N_p}{N_r} \quad (19)$$

which is now situated below the airplane. The motion is like that of a pendulum placed at this height above its point of support.

TWO-CONTROL OPERATION IN STEADY TURNS

The two-control average airplane, showing stability both in combined yawing and sideslipping (rolling control) and in combined rolling and sideslipping (yawing control), should reach a definite condition of equilibrium with some fixed setting of the lateral control. In general, the equilibrium condition corresponding to a definite rudder or aileron setting will be a steady turn at a definite angle of bank. If the components of rolling and yawing angular acceleration produced by the deflected controls are δL_s and δN_s , as before, the equations of lateral equilibrium at a fixed angle of bank may be written:

$$\left. \begin{aligned} g\varphi - rU_0 + vY_r &= 0 \\ rL_r + vL_s + \delta L_s &= 0 \\ rN_r + vN_s + \delta N_s &= 0 \end{aligned} \right\} \quad (20)$$

In case control is by ailerons giving secondary (adverse or favorable) yawing moments, the term N_s is replaced by κL_s ; and, in case control is by rudder alone, L_s is dropped from the equations. In any case it has

¹ This mode of oscillation has been discussed by Lanchester.

to be assumed that the longitudinal control is properly manipulated for maintaining altitude and speed while turning.

Two special conditions of equilibrium are of interest. Solving the equations for the angle of bank

$$\varphi = \frac{(Y_r L_r + L_r U_0) \delta N_s - (Y_s N_r + N_s U_0) \delta L_s}{g(L_r N_s - L_s N_r)} \quad (21)$$

The necessary condition for the bank angle to be zero with deflected controls is:

$$\frac{1}{\kappa} = \frac{L_s}{N_s} = \left(\frac{Y_s L_r + L_s U_0}{Y_r N_r + N_s U_0} \right) \quad (22)$$

(See equation (14).)

In case the applied control rolling and yawing moments are in this ratio, the steady state of motion of the airplane will be a flat turn without bank. This limiting ratio may be compared with the ratio of the secondary aileron yawing moments to the rolling moments. If the secondary moment is adverse and exceeds a certain proportion of the rolling moment, an equilibrium condition in which the ailerons do not produce a bank of the airplane becomes possible. In this condition a gradual deflection of the ailerons would merely cause the airplane to assume a yawed attitude, turning slowly under the influence of the side pressure vY_s . Such a condition should be especially avoided in a two-control airplane utilizing aileron operation.

Another simpler condition of equilibrium that is also of interest is the condition for zero rate of yawing with deflected controls. The resolution of the equations in this case is:

$$r = \left(\frac{L_s N_s - N_s L_s}{L_r N_r - L_r N_r} \right) = 0 \quad (23)$$

This is the condition for an ordinary sideslip and the ratio of yawing to rolling moment requisite to this condition is simply

$$\frac{1}{\kappa} = \frac{L_s}{N_s} = \frac{L_r}{N_r} \quad (24)$$

Obviously it should be considered undesirable to allow the secondary adverse yawing moment of the ailerons to approach this proportion of the rolling moment.

By a similar resolution of the equations another condition, namely,

$$\frac{1}{\kappa} = \frac{L_s}{N_s} = \frac{L_r}{N_r} \quad (25)$$

is obtained for the case of steady turning without sideslipping. This equilibrium is possible with aileron control alone in the case of secondary adverse yawing moments and furnishes another criterion for the magnitude of these secondary moments. In this case it would be expected that a gradual application of the rolling control would lead to turning at a progressively

greater rate with the angle of bank opposite in sense to the applied rolling moment.

The main point of interest in the condition of steady turning with two-control operation is the angle of sideslip incident to the turn at various angles of bank. The resolution of the equations for v results in:

$$v = g \varphi \frac{N_r - \kappa L_r}{\kappa (Y_s L_r + U_0 L_s) - (Y_r N_r + U_0 N_s)} \quad (26)$$

In the case of rudder control, where $L_s = 0$ the expression for v reduces to:

$$v = \frac{-g \varphi}{\left(Y_s + U_0 \frac{L_s}{L_r} \right)} \quad (27)$$

while in the case of pure rolling-moment control (ailerons giving no secondary yawing moments)

$$v = \frac{-g \varphi}{\left(Y_s + U_0 \frac{N_s}{N_r} \right)} \quad (28)$$

Thus the sideslip incident to turning with only rudder control is mainly dependent on the ratio of L_s/L_r , while with rolling-moment control the important factor is

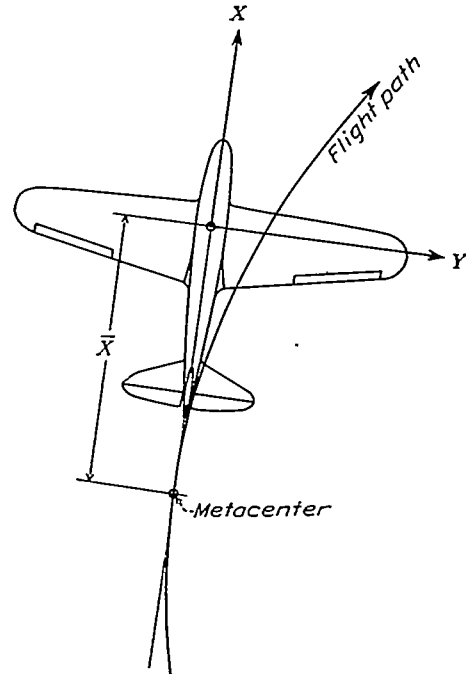


FIGURE 1.—Diagram illustrating combined yawing and sideslipping motion during a steady two-control turn. Metacenter for yawing moment $\bar{x}_N = \frac{N_r}{N_s}$; metacenter for rolling moment $\bar{x}_L = \frac{L_r}{L_s}$.

N_s/N_r . In both cases the sideslip will ordinarily be positive (toward the center of the turn) although the airplane does not necessarily lose altitude on this account.

Figure 1 illustrates the combined sideslipping and yawing of a two-control airplane during a steady turn.

In the case of rudder control the inward sideslip must be such that $vL_r = -rL_r$ to prevent rolling. This combined sideslipping and yawing motion may be ascribed to a rotation of the airplane about some point aft of the center of gravity. If the distance of this point behind the center of gravity is denoted by \bar{x}_L

$$r\bar{x}_L L_v = -rL_r$$

$$\text{or} \quad \bar{x}_L = -\frac{L_r}{L_v} \quad (29)$$

for the case of rudder-controlled turns. For rotation of the airplane about this point the rolling moment vanishes, hence the point is a metacenter for the rolling moment. The X axis will be tangent to the flight path at this point in rounding a turn, as shown in figure 1.

Similar considerations apply in the case of operation with a rolling-moment control with fixed rudder. Here the metacenter is for a vanishing yawing moment, the amount of sideslip being that necessary for $vN_v = -rN_r$. The distance of the metacenter aft of the center of gravity is found from

$$r\bar{x}_N N_v = -rN_r$$

$$\text{or} \quad \bar{x}_N = -\frac{N_r}{N_v} \quad (30)$$

An interesting point arises in connection with the relation of the two metacenters (\bar{x}_L and \bar{x}_N). For positive rotation of the airplane about a point nearer the center of gravity than \bar{x}_N the residual yawing moment will be negative; hence if the metacenter \bar{x}_L is nearer the center of gravity than \bar{x}_N , steady turning with rudder operation will require a positive setting of the rudder, i. e., in a direction to aid the turn. Conversely, if control is by rolling moments, the steady motion will be a rotation about \bar{x}_N and, if the residual rolling moment for rotation about this point is negative ($\bar{x}_L < \bar{x}_N$), the rolling control setting will be positive, also in a sense aiding the turn. Obviously, the condition $\bar{x}_N < \bar{x}_L$ corresponds to *instability* since in this case with either mode of two-control operation the control setting during a steady turn would be one appropriate to recovery from the turn. This condition is analogous to the spiral instability discussed by Lanchester. The following table gives the metacenters \bar{x}_L and \bar{x}_N for the average airplane at various lift coefficients:

C_L	\bar{x}_L	\bar{x}_N
	Feet	Feet
0.35	20	25
1.0	45	29
1.8	55	41

At the lowest speeds ($C_L = 1.0$ and 1.8) \bar{x}_N is less than \bar{x}_L , indicating that negative rudder and aileron settings will be required during steady positive turns. Figure 2 shows results of calculations of the control-moment coefficients for equilibrium in turning at

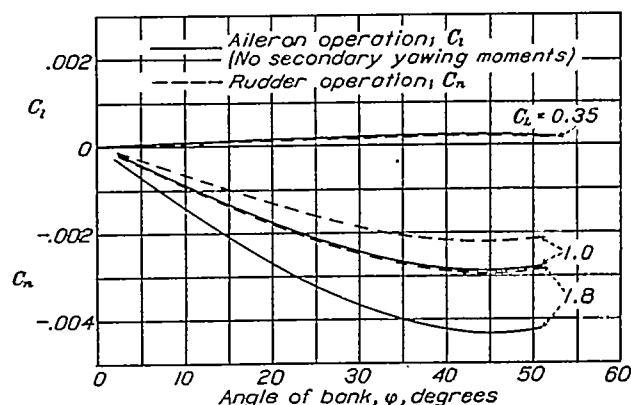


FIGURE 2.—Moment coefficients indicating control settings during steady turns at various angles of bank.

various angles of bank that give an indication of the fixed control settings.

Equilibrium angles of sideslip in steady turning with both modes of two-control operation are shown in figure 3. It is to be noted that the angle of sideslip is not greatly different in steady turning with either type of control and in every case is positive.

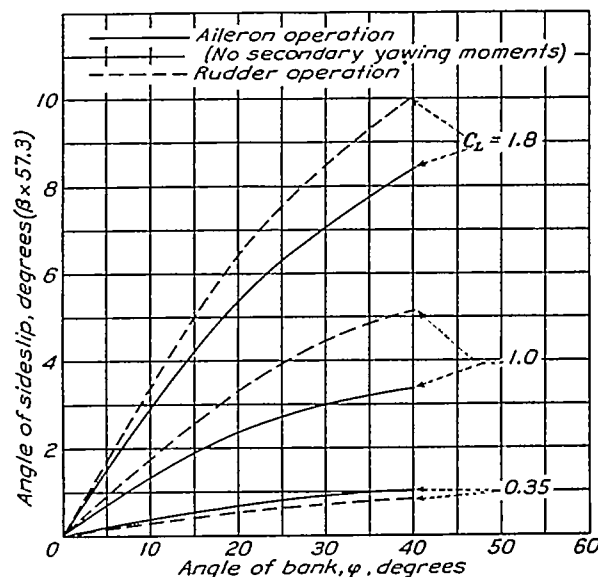


FIGURE 3.—Angles of sideslip during steady turns at various angles of bank with different modes of two-control operation.

The only possibility of outward or negative sideslip during the steady turn occurs when rolling and yawing moments are applied in combination. Such an occurrence is illustrated in figure 4, which shows the effect of secondary aileron yawing moments on the equilibrium during 30° bank turns. At $C_L = 1.0$ the sideslip becomes

negative, or outward, when the ratio N_s/L_s exceeds negatively N_r/L_r , i. e.:

$$\kappa = \frac{C_n}{C_l} \frac{I_x}{I_z} = \frac{N_s}{L_s} > \frac{N_r}{L_r} \quad (31)$$

(See equation (25).)

Whether or not a given secondary aileron yawing moment will reduce or increase the equilibrium sideslip angle during a steady turn depends on the spiral stability of the airplane, for this characteristic determines the sign of the equilibrium control setting.

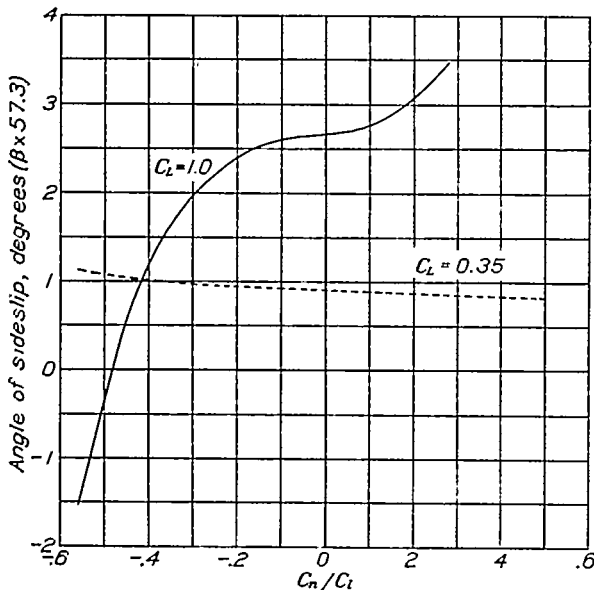


FIGURE 4.—The effect of secondary yawing moments on sideslip during a 30° bank steady turn; two-control operation with ailerons.

Thus, in the case of a spirally unstable machine the aileron setting will be appropriate to recovery from the bank and an adverse yawing moment will act in a positive direction, aiding the turn. In any event, spiral stability, if present, must be considered as a small effect (with conventional airplanes); and the control setting during steady turns is, if positive, almost certain to be small so that secondary moments will have little effect. (See fig. 4, $C_L = 0.35$.)

TWO-CONTROL OPERATION IN UNSTEADY TURNS

The consideration of the equilibrium state is sufficient for the study of conditions during slowly executed maneuvers of sufficient duration for the natural free oscillations of the airplane to die out. In the case of rapid maneuvers performed by more or less quick movements of the control the equilibrium conditions are of secondary importance and the primary consideration is the oscillation and damping of the free motion.

According to the previously outlined treatment, the motions of the two-control airplane set up during unsteady turns will be studied by considering a constraint impressed on the motion in the particular coordinate in which the available control operates. Thus in one case of rudder control a definite sequence of yawing motions appropriate to the turn maneuver under consideration will be assumed. The free rolling motion that the airplane takes up during the maneuver will then be studied and compared with the rolling motion that would be considered appropriate for the execution of the maneuver.

The investigation of unsteady conditions during various maneuvers required that the equations of motion (equations (1) to (3)) be solved for different types and variations of the impressed disturbances. The first step in the procedure consisted in obtaining solutions of the equations for "unit disturbances" substituted into each coordinate of freedom.

The unit disturbance is defined by

$$\begin{cases} 1(t) = 0 & \text{when } t < 0 \\ 1(t) = 1 & \text{when } t > 0 \end{cases} \quad (32)$$

(see reference 3) and is taken to represent a disturbing acceleration of unit magnitude applied instantly at $t=0$.

The solutions of the equations of motion for this type of disturbance were found by methods described in reference 4. The result thus obtained is analogous

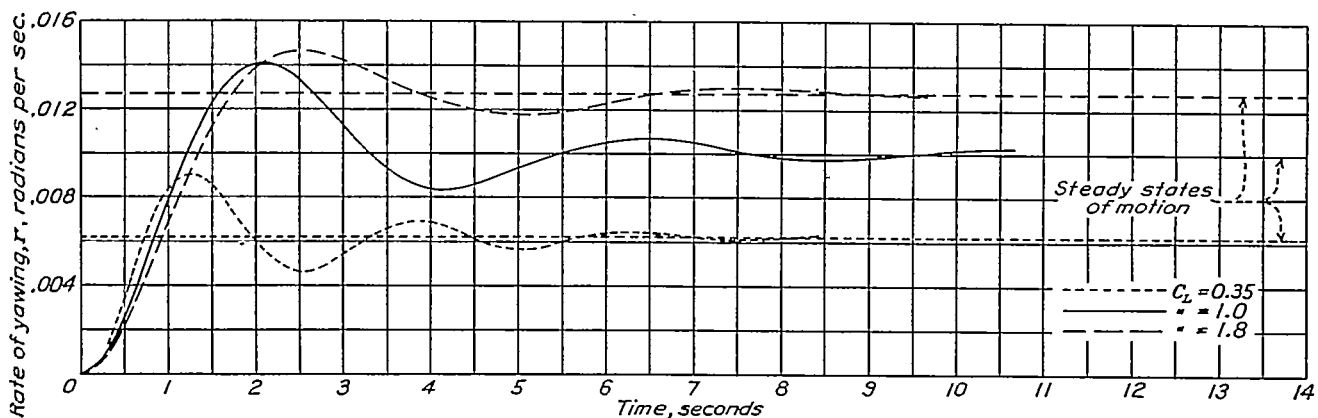


FIGURE 5.—Yawing motion due to unit side disturbance; two-control airplane constrained in rolling (aileron operation; $\kappa=0$).

to the so-called "indicial admittance" of the electric-circuit theory and was combined with Carson's generalized expansion theorem (see reference 5) to obtain the motion due to the varying forms of disturbance. If $v_1(t)$ is the motion calculated for a unit disturbance $1(t)$, and $v(t)$ is the motion due to a varying distur-

As the curves show, the actual yawing is delayed for an instant but in each case oscillates about the mean value given by (35). The most favorable condition is that at high speed ($C_L=0.35$) since the appropriate yawing motion occurs with the least delay and the oscillations are most quickly damped.

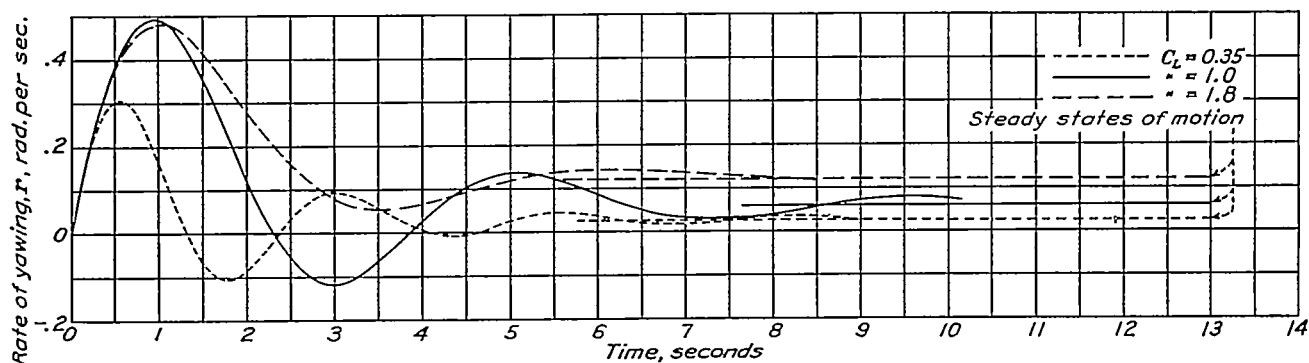


FIGURE 6.—Yawing motion due to unit yawing disturbance; two-control airplane constrained in rolling (aileron operation; $\kappa=0$).

ance, say $\varphi(t)$ (see equation (2)), then Carson's theorem may be written

$$v(t_n) = v_1(t_n)\varphi(0) + \int_0^{t_n} v_1(t_n-t) \frac{d\varphi}{dt} dt \quad (33)$$

It was found convenient to evaluate this integral graphically.

Figures 5 and 6 show the motions of the two-control airplane constrained in rolling (aileron operation) due to unit disturbances acting in each of the two remaining

As stated previously, the unit motions, or motions due to unit disturbances, were utilized in calculating the effects of varying disturbances assumed during turn maneuvers. Thus the curves given in figure 5 were used to find the motions due to a varying angle of bank by means of Carson's integral (33). Actually, in constraining the airplane to a definite bank angle as was assumed, a varying aileron rolling moment has to be applied and, if this moment is accompanied by a

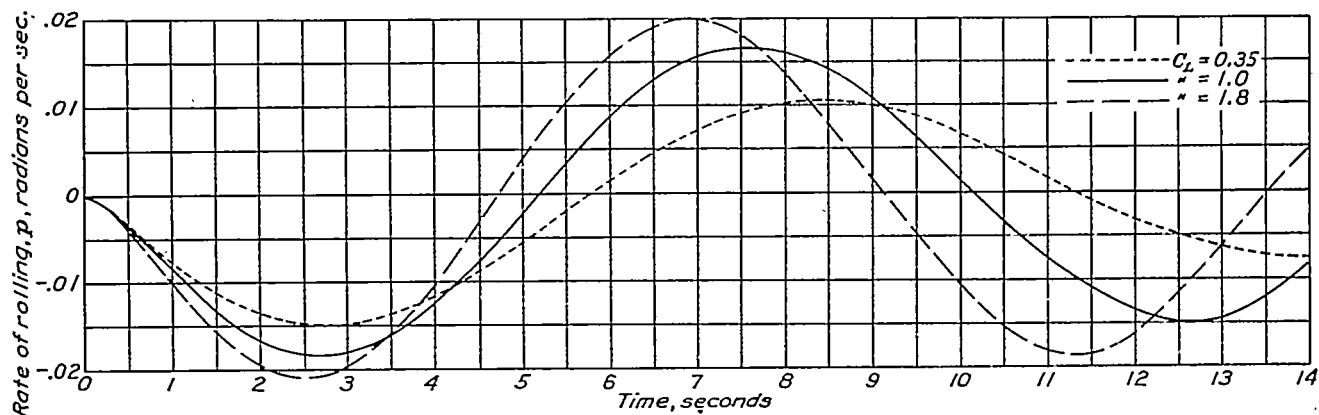


FIGURE 7.—Rolling motion due to unit side disturbance; two-control airplane constrained in yawing (rudder operation).

degrees of freedom. Figure 5 shows the yawing motions resulting from a suddenly impressed sidewise acceleration of 1 foot per second per second. The conditions here may be assumed to represent the effect of an initial and constantly maintained angle of bank of approximately

$$\varphi = \frac{1}{g} \quad (34)$$

In order to maintain this bank angle without sideslipping, the airplane should immediately acquire a uniform rate of yawing of approximately

$$r = \frac{1}{U_0} \quad (35)$$

secondary yawing moment, additional disturbances in yawing will be introduced. The rolling motion will also introduce a secondary disturbance in yawing equal to $N_p \times p(t)$. Figure 6 shows the yawing motion produced by a unit disturbance in yawing that was used in calculating the effects of such impressed yawing disturbances. This curve may be considered to represent the yawing motion following the sudden application of a control yawing moment. The final effect of this disturbance is to cause the machine to assume a yawed attitude, turning slowly under the influence of the side force vY .

Figures 7 and 8 show the corresponding solutions of the equations of motion (3) for the case of the airplane constrained in yawing by a rudder control. Figure 7

may be taken to represent the rolling motion following an initial bank angle. Presumably the ideal condition would be a rapid diminishing of this bank angle to zero. The integrated areas under the curves shown would then approach a definite value after a few oscillations, which area should be equal to the initial bank angle, namely approximately

$$\varphi = \frac{1}{g} \quad (36)$$

Instead, the airplane continues to roll one way and then the other, executing the pendulum-like oscillations

followed in practice. In other respects, it was thought that any smooth curve representing the banking or yawing of the machine up to a definite angle or rate maintained steadily for a short time and followed by a smooth recovery to straight flight would serve the purpose. Figure 9 shows the time history of the ideal three-control turn that was assumed in the subsequent investigation. In most cases the maneuver was assumed to be completed in 6.28 seconds and this time is taken to represent about the maximum rapidity with which the maneuver could be performed at the lowest speed using conventional-type controls. Figure 10

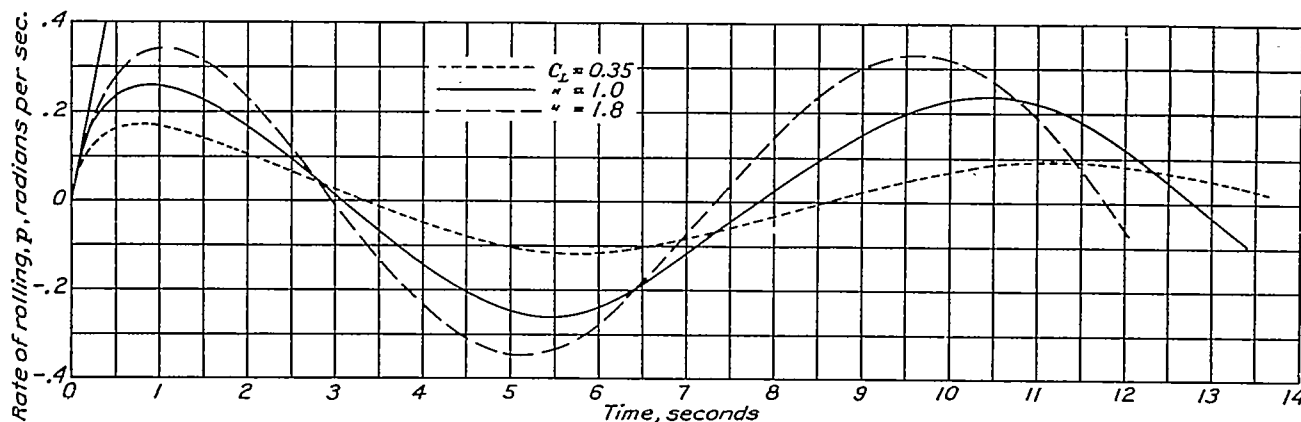


FIGURE 8.—Rolling motion due to unit rolling disturbances; two-control airplane constrained in yawing (rudder operation).

described in the discussion of the stability of this motion. The damping of these oscillations is slight and is most apparent at the lowest lift coefficient, $C_L = 0.35$.

Figure 8 is similar to figure 7 except that here the rolling motion is due to a suddenly impressed angular acceleration in rolling. These curves were used in calculating the effect of varying rolling moments impressed indirectly by yawing motion $L_r \times r(t)$. (See equation (3).) Figure 8 is of interest in illustrating the two more or less distinct modes of motion in free rolling and sideslipping. It will be noted that the rolling starts very rapidly (with an initial angular acceleration of one radian per second per second) but soon takes up the slow swinging oscillation. As in the previous case of rolling motion, the steady state finally approached is a definite angle of bank.

The foregoing calculations are of interest in indicating how the different types of two-control airplanes may be expected to respond to attempted maneuvers. The first step in the calculation of an actual complete maneuver is to arrive at a specification for that part of the motion which is assumed to be constrained. It will be of interest to compare the motions executed by the two-control airplane with the most perfect possible coordination of the motions that might be obtained with three-control operation. Obviously, it will be necessary to specify a maneuver that is within the power of the control to produce and it will be desirable to conform the specification to a type of turn likely to be

shows the control-moment coefficients necessary to constrain the rolling and yawing motions to the specified maneuver with perfect three-control operation. Under the conditions of two-control operation the turns

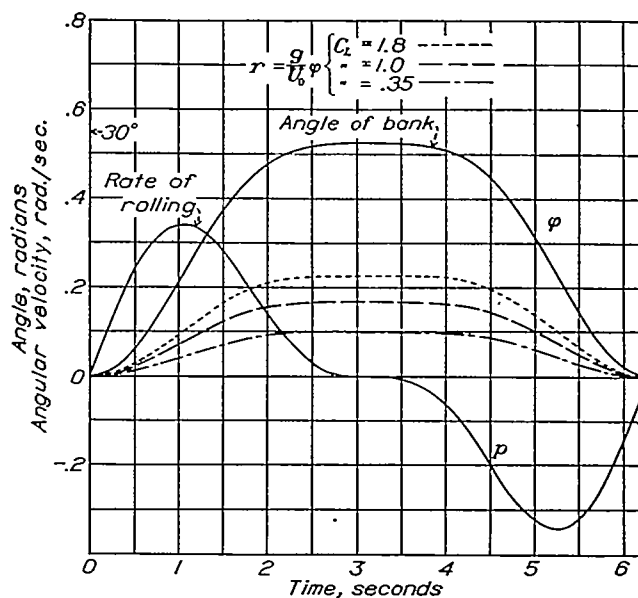


FIGURE 9.—Angle of bank and rates of rolling and yawing specified for 30° bank two-control turn maneuvers.

will not be perfect owing to the sideslipping and it is to be expected that this sideslipping will in some degree modify the control settings.

In the calculations illustrated in figure 11 the banking motion was assumed to be forced to follow the ideal bank by means of a rolling control and the resultant free yawing motions were computed. The reaction of the machine was evidently favorable in this case. This result could have been anticipated from the calcu-

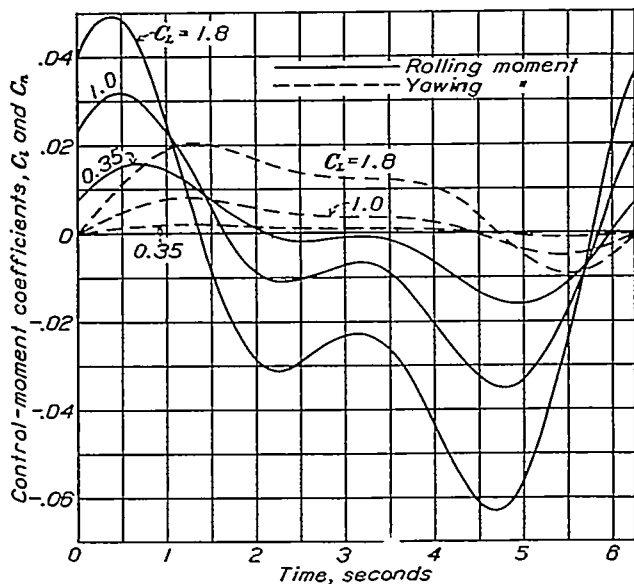


FIGURE 10.—Control-moment coefficients necessary to produce specified maneuver with zero sideslip.

lations of stability, which showed that the free yawing motion was of short period and strongly damped.

The curves of figure 11, although indicating the advantage of rolling-moment control, also bring out an imperfection in the coordination of the yawing motion. The rolling motion itself tends to induce an unfavorable

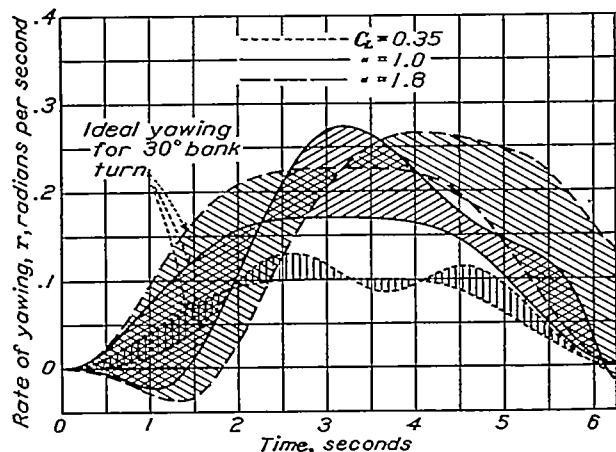


FIGURE 11.—Free yawing motion during 30° bank maneuvers performed with rolling control.

yawing motion at the start of the maneuver due to the adverse sign of N_p . This effect becomes more pronounced at the higher lift coefficients and, in the worst case ($C_L=1.8$), produces an adverse change in the heading of the machine of 2.0°. The total change in heading produced by the maneuver at this speed is approximately 50°.

From the foregoing considerations, it appeared that a certain amount of favorable secondary aileron yawing moment might be desirable to overcome the adverse yaw caused by the rolling motion at the start of the

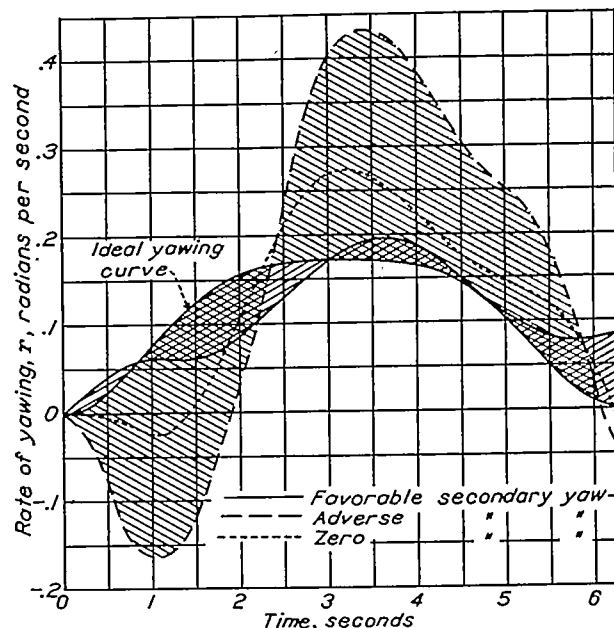


FIGURE 12.—The effect of secondary yawing moments on yawing motions during 30° bank maneuver performed with rolling control; $C_L=1.0$. $C_n=\pm 0.21C_l$ ($\alpha=\pm 0.15$).

turn. The effects of secondary yawing moments of both favorable and adverse sign applied in proportion to the control rolling moment are illustrated in figures 12, 13, and 14.

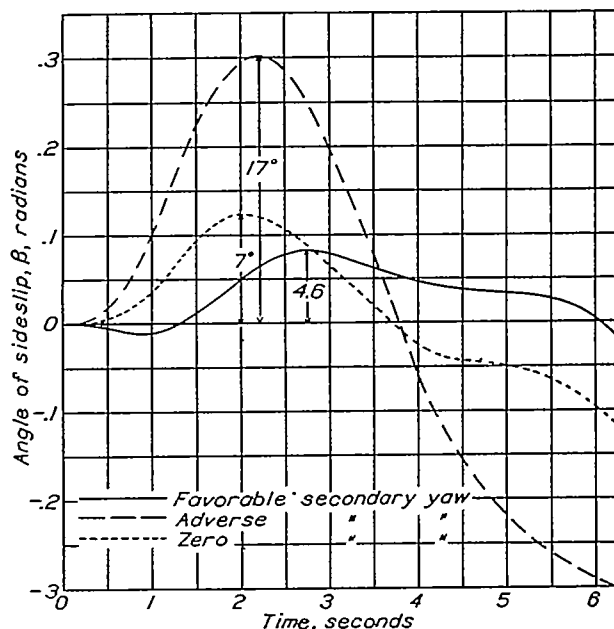


FIGURE 13.—The effect of secondary yawing moments on sideslip during 30° bank maneuver performed with rolling control; $C_L=1.0$. $C_n=\pm 0.21C_l$ ($\alpha=\pm 0.15$).

The curves shown were calculated by equation (2) and take account of the increments of control displacement necessary to accommodate the rolling moments introduced by the yawing and sideslipping oscillations.

The effect of these increments of control displacement is to modify the stability of the yawing and sideslipping motions, an adverse yawing moment reducing the damping and lengthening the period. The results indicate especially the disadvantage of adverse yaw and show that some improvement may be had from a favorable yawing moment.

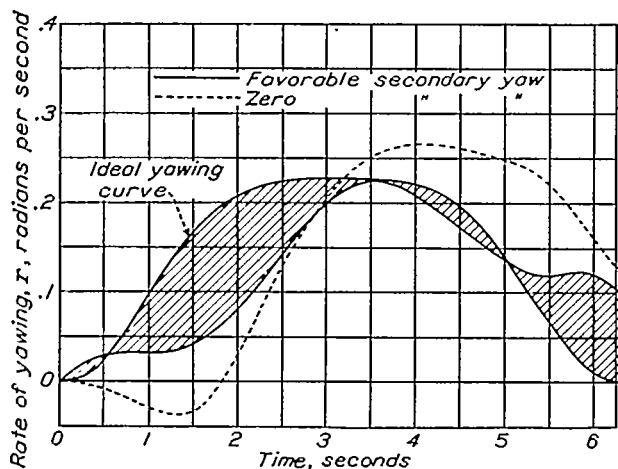


FIGURE 14.—The effect of favorable secondary yawing moment on yawing motion during 30° bank maneuver performed with rolling control; $C_L=1.8$. $C_n=0.21C_l$ ($\kappa=0.15$).

In order to study more closely the possible beneficial effects of a favorable aileron yawing moment, it is of some interest to analyze further the control application into several components. The component that results in modification of the stability through the action of the secondary yawing moment may be considered to be directly favorable to improved coordination of the

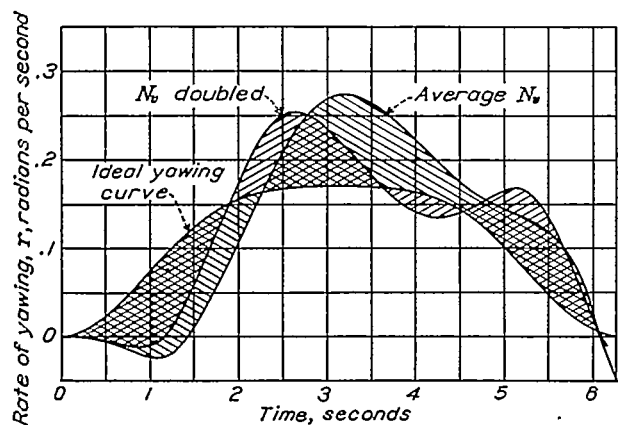


FIGURE 15.—The effect of increased N_v on yawing motion during 30° bank maneuver; aileron operation (no secondary yawing moment); $C_L=1.0$.

yawing motion because it shortens the natural oscillation period and increases the damping. With a given proportion of favorable yawing moment, increasing the dihedral angle should result in further improvement in this respect since the apparent weathercock stability ($N_v - \kappa L_p$) is increased in that way. Another component of the applied rolling control is directed to overcoming the damping of the rolling incident to the maneuver. The secondary yawing disturbance thus

introduced is of the same form as pN_p and may be calculated as

$$N_p' = (N_p - \kappa L_p) \quad (37)$$

The condition for perfect coordination of banking and yawing motion during the turn requires that the acceleration in yawing be very nearly proportional to the rate of rolling; namely,

$$\frac{dr}{dt} = \frac{g}{U_0} \times p \quad (38)$$

The component of rolling control directed toward opposing the damping in rolling is applied in this way

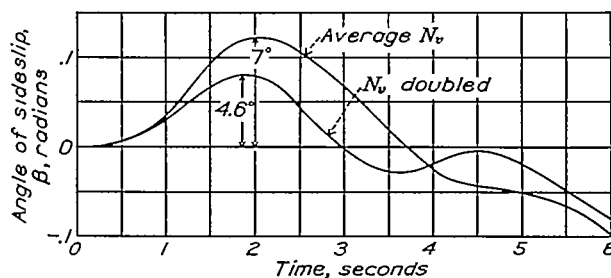


FIGURE 16.—The effect of increased N_v on sideslip during two-control 30° bank maneuver; aileron operation (no secondary yawing moment); $C_L=1.0$.

and it is seen that this component of the secondary favorable yawing moment is properly directed toward improved coordination of the yawing motion. The component of control application necessary to accelerate the rolling motion does not, however, lead to a desirable secondary yawing acceleration since this acceleration is not proportioned to the rolling velocity.

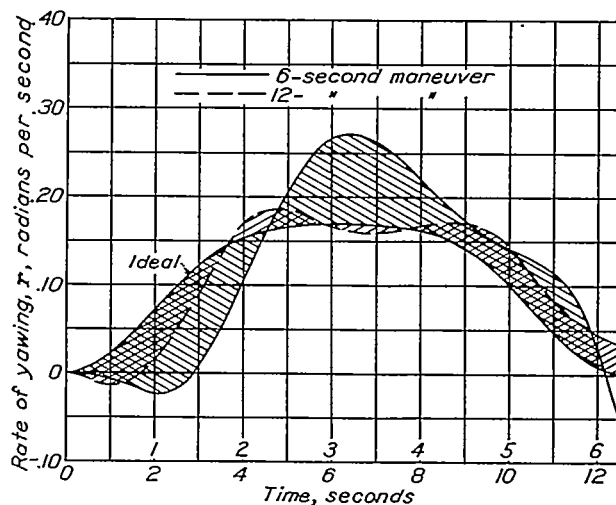


FIGURE 17.—Comparison of yawing motions during maneuvers of different time extents; aileron operation (no secondary yawing moment); $C_L=1.0$.

This component results in the primary disadvantage associated with favorable-yaw ailerons. Quick or irregular movements of the control may lead to pronounced yawing oscillations if the secondary moment is very great.

It appears that a decisive method of improving the aileron-operated two-control airplane would be to

increase the weathercock-stability factor N_z . This method would serve directly to reduce the sideslipping to a minimum both in steady turning and in rapidly executed turn maneuvers. Figure 15 shows the effect of doubling N_z on the yawing motion during the maneuver performed at $C_L=1.0$. This modification of the airplane shortened the natural period of the oscillation and resulted in the yawing action taking place more quickly. The effect on sideslip is shown in figure 16. Although the maneuver ends with about 5° of outward sideslip, this value will be quickly reduced to zero on account of the natural stability of the motion. With different timing of the maneuver it may, of course, be brought to an end with no residual sideslip. The following table shows the effect of arbitrarily increasing N_z on the natural period of the yawing oscillations:

Ratio of N_z to that of average airplane	Period
1	Seconds 4.16
2	2.92
4	2.05

It is to be noted that an increase in vertical-fin area will increase the derivative N_z as well as N_y and will thus result in greater damping of the motion.

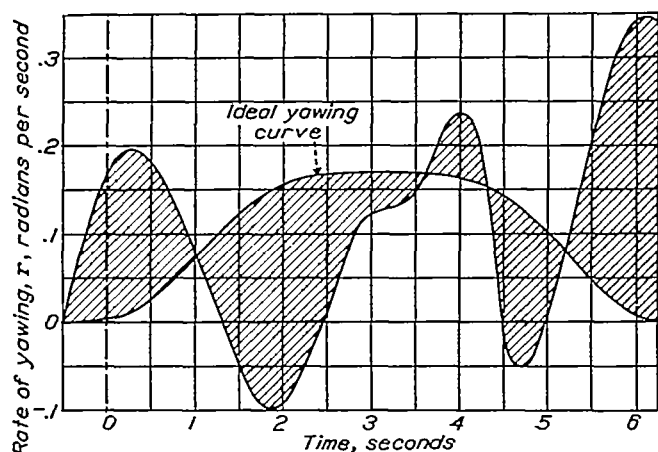


FIGURE 18.—Yawing motion necessary to enforce assumed 30° bank maneuver with rudder operation; $C_L=1.0$.

A certain disadvantage associated with increased N_z is the relatively greater tendency for spiral instability and the consequent necessity for holding the control against the steady turn. It may be expected, however, that this undesirable tendency could be overcome by properly proportioning the dihedral of the wings. The greatest possible effect of increase of vertical-fin area would be to cause the metacenter for yawing moments \bar{x}_N (see discussion of stability) to approach coincidence with the fin; it would then appear necessary to arrange the metacenter for rolling moments ahead of this point in order to accommodate any desired increase of vertical-fin area and secure spiral stability.

Further improvement in the operation of the aileron-controlled machine could be had by decreasing the yawing derivative in rolling N_p . Alteration of this derivative apparently would require fundamental changes in wing design, improvement being in the direction of lower aspect ratio, which might, of course, conflict with other requirements.

As pointed out, the maneuvers assumed in these calculations are considered to be more rapid than usual in

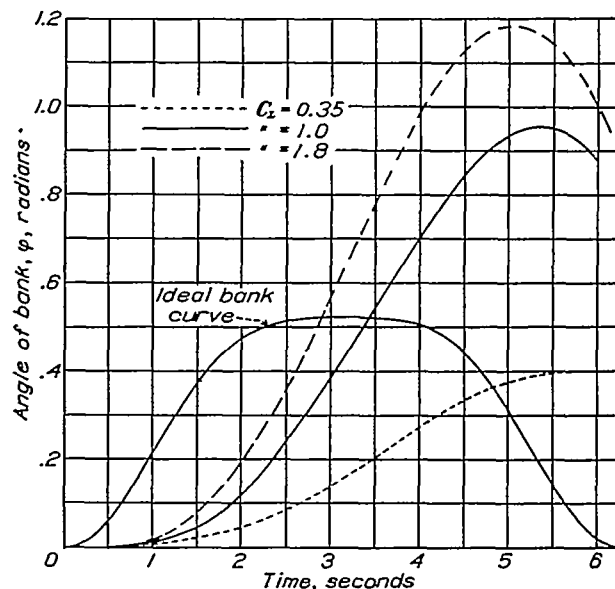


FIGURE 19.—Free angles of bank during turn maneuvers performed with rudder compared with ideal bank curve; yawing constraints for 30° bank maneuver.

normal flight, since they represent the use of a large proportion of the control power ordinarily available at the lower speeds. With slower maneuvers the coordination of the motions of the two-control airplane would be expected to be much better, especially when the duration of the maneuver becomes large relative to the natural period of oscillation of the airplane. Figure 17 shows the result of a calculation in which the duration of the 6.28-second maneuver was doubled.

It is worth noting that the actual deflection of the flight path of an airplane relative to the earth is accomplished much more directly by banking than by steering. Regardless of the sideslipping and coordination of angular motions, any decided acceleration of the path must be brought about by inclination of the lift and is not directly affected to any great extent by rotating the airplane in yaw. Such deflection of the path would be the principal objective in turning to avoid an obstacle. Thus the airplane with rolling-moment control should be capable of avoiding obstacles equally as quickly as a conventional three-control airplane. As is the case with three-control operation, the tendency of a two-control airplane to accelerate downward when banked must be counteracted by a movement of the elevator.

If the airplane is assumed to execute a sharp turn to avoid an obstacle, the primary consideration will thus be the ability to produce a specified bank. Under such

conditions the pilot of the rudder-operated airplane would be expected to make an effort at indirect control of the bank without regard to the coordination of the yawing motion. The question then arises as to what yawing motion would have to be prescribed in the case of the rudder-controlled machine to enforce the desired motion in banking.

Figure 18 shows the yawing motion that results in a bank curve similar to that given in figure 9. It appears that, in order to attain the bank angle as shown, a relatively powerful rudder control would have to be applied about one-half second in advance of the usual start of the turn. Further calculations showed that the prescribed yawing motion could be attained throughout if a rather large amount of rudder control were avail-

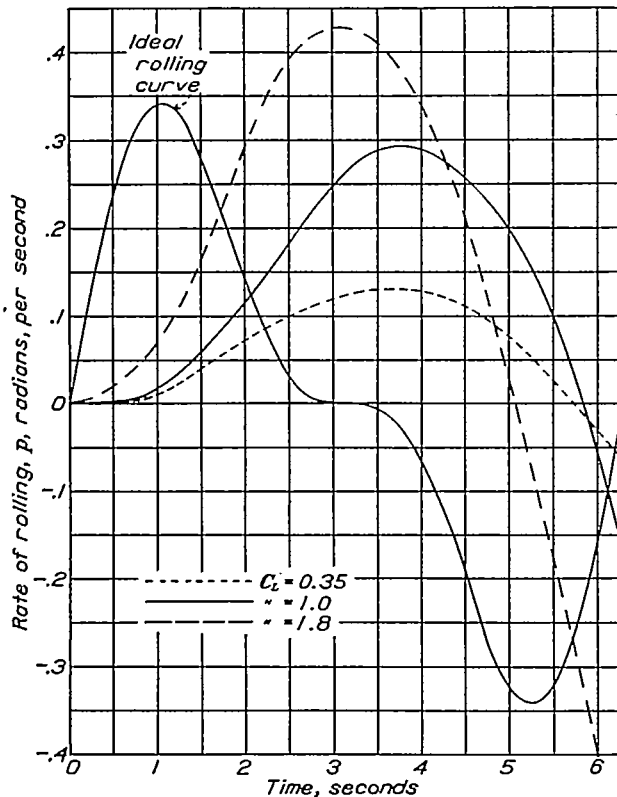


FIGURE 20.—Free rolling motion during turn maneuvers performed with rudder; yawing constraints for 30° bank maneuver.

able. That such an attempt to follow a definite course in banking would require a vigorous use of the rudder is evident from the oscillation of the yawing curve.

In the case of two-control operation with a constraint in yawing by means of the rudder, the yawing motions shown in figure 9 were assumed and the resulting free rolling motions were calculated. Figures 19 and 20 show the results of such calculations made at different lift coefficients. The angles of bank and rates of rolling attained are compared with those that would be appropriate to the constrained yawing motion. It is apparent from these and the preceding figures that the two-control airplane operated with the rudder cannot be expected to perform rapid maneuvers of the type considered. The natural reaction of the rolling

motion is too slow and the damping is too slight to enable even an approximate coordination of the motions within the short time of duration of the maneuver.

Figure 21 shows the angles of sideslip attained with the various modes of operation considered, summarizing the results of the calculations.

The reasons for the inability of the rudder-controlled airplane to execute rapid turns are: First, that the secondary rolling reaction due to yawing motion is insufficient to overcome the relatively great damping of direct rolling motion; second, that for a rapid turn the rate of rolling required on entry and recovery greatly exceeds the maximum rate of yawing; and third, that the free rolling and sideslipping oscillations set up are not very well damped. The greatest possibility for

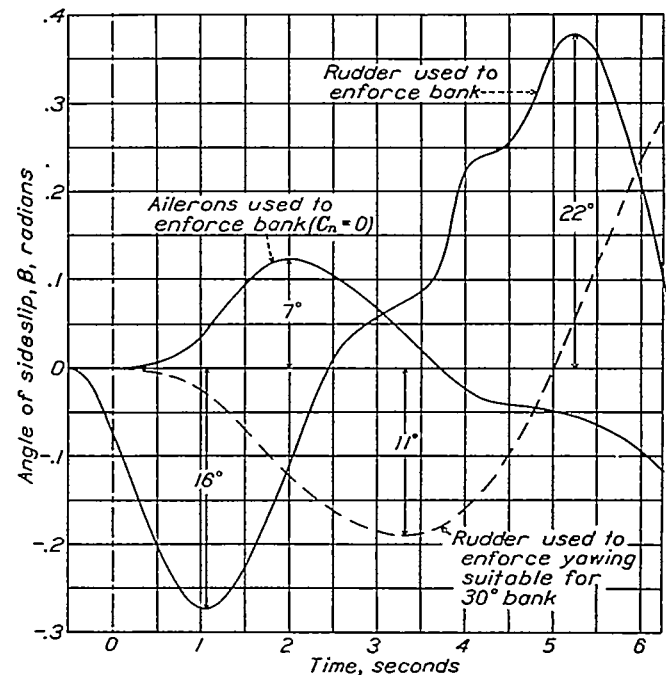


FIGURE 21.—Angles of sideslip during two-control turn maneuvers with different modes of operation; 30° bank turn maneuver.

improvement would appear to be in increasing the derivatives L_r and Y_r . The first (L_r) would call for increased dihedral angle and would serve to shorten the natural period of the rolling and sideslipping motion, while the second (Y_r) would call for increased area of the side projection of the airplane and should improve the damping of the oscillations. The following table shows the effects of changing these derivatives on the natural period and damping of the oscillations at $C_L = 1.0$.

	Ratio of derivative to that of average airplane				
	L_r			Y_r	
	$\frac{1}{2}$	1	2	1	2
Time to damp $\frac{1}{2}$, seconds.....	12.4	36.6	∞	36.6	6.25
Period, seconds.....	15.3	9.9	7.65	9.9	10.9

CONCLUSION

The lateral motion of a conventional airplane is more stable when constrained in rolling than when constrained in yawing. The stability of the free yawing and sideslipping motion is greater than that of the entirely free motion; the stability of the free rolling and sideslipping is less than that of the entirely free motion.

If a rolling-moment control is used to enforce an arbitrary constraint in banking, the free yawing that results will be approximately coordinated to the bank if the airplane has the average degree of weathercock stability (N_z). The yawing in this case is also approximately adjusted to the speed of flight so that with a given bank maneuver a more rapid rate of yawing is attained at low speed than at high speed, as is desirable. The deviation of the yawing from the ideal is greater, however, at lower speeds and is also greater in quick turns than in more slowly executed ones. If the rolling control were designed to give a moderate favorable yawing moment, the coordination of the motions would be improved. Improvement may also be effected by increasing the weathercock stability. If, however, the aileron control gives the usual proportion of secondary adverse yawing moment, the coordination of the yawing with the banking will be relatively very poor. The motions may then become unstable and uncontrollable in an extreme case at high lift coefficient. These latter statements are particularly applicable to conventional-type ailerons, which are considered as undesirable on this account for use at low flight speed unless compensated by the rudder.

A rudder control may be used to enforce a constraint either directly on the yawing motion or indirectly on the rolling motion provided that the maneuver specified is not too rapid nor the disturbances encountered too severe. In the former case the free banking motion occurs as a series of long oscillations that do not begin to approximate the desired bank until some time after the start of a maneuver or after the passing of a disturbance. During a rapid yawing maneuver the bank that occurs is greater at low flight speed than at high, indicating that the coordination of the centrifugal and the gravitational accelerations is not adapted to the desired variation with flight speed.

Although the coordination of the motions with aileron control grows worse as the flight speed is re-

duced, the coordination with rudder control improves somewhat at the lower speeds. This effect would be especially apparent if the rudder were applied in such a way as to enforce indirectly a desired banking motion. Such indirect control requires, however, that the rudder be deflected in advance of the desired effect. The yawing that arises when the bank is indirectly controlled with the rudder is a very poor approximation to the ideal yawing and calls for large and irregular control movements.

The amount of sideslipping during *steady* turns is not greatly different with either mode of operation. In either case it appears desirable that the free motion of the airplane show spiral stability so that control settings opposing the turn will not be required.

In general, it is concluded that a reliable rolling-moment control that does not give a secondary adverse yawing moment would afford the most satisfactory means for two-control operation. It appears that a moderate amount of favorable secondary yaw would be desirable although certain disadvantages appear if the proportion is too great.

The disadvantage in two-control operation lies not so much in the imperfection of control of the flight path of the airplane relative to the earth as in the sideslipping and sidewise accelerations that arise through the imperfect coordination of the yawing and banking motions. It appears possible that this tendency may be so reduced by the use of suitable control organs and properly modified stability characteristics as to be unobjectionable.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., August 12, 1936.

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